OCR
Oxford Cambridge and RSA

AS Level Mathematics A

H230/01 Pure Mathematics and Statistics

Question Set 3

1 It is given that $\mathrm{f}(x) = 3x - \frac{5}{x^3}$.

Find

(a) $\mathrm{f}'(x)$, **[3]**

(b) $\mathrm{f}''(x)$, **[2]**

(c) $\int \mathrm{f}(x)\,\mathrm{d}x$. **[3]**

2 The circle $x^2 + y^2 - 4x + ky + 12 = 0$ has radius 1.

Find the two possible values of the constant k. **[4]**

3 **In this question you must show detailed reasoning.**

(a) The polynomial $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = 2x^3 + 3x^2 - 8x + 3$.

(i) Show that $\mathrm{f}(1) = 0$. **[1]**

(ii) Solve the equation $\mathrm{f}(x) = 0$. **[4]**

(b) Hence solve the equation $2\sin^3\theta + 3\sin^2\theta - 8\sin\theta + 3 = 0$ for $0° \leqslant \theta < 360°$. **[5]**

4 **(a)** Find the coordinates of the stationary points on the curve $y = x^3 - 6x^2 + 9x$. **[4]**

(b) The equation $x^3 - 6x^2 + 9x + k = 0$ has exactly one real root.

Using your answers from part **(a)** or otherwise, find the range of possible values of k. **[2]**

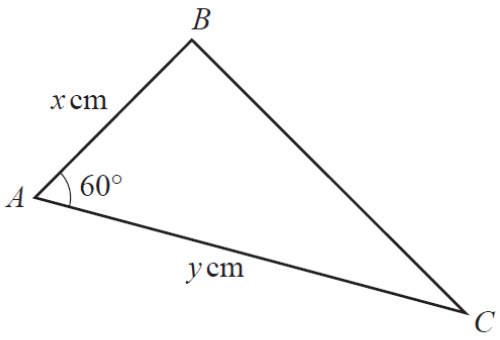
5 **(a)** Prove that the following statement is **not** true.

m is an odd number greater than $1 \Rightarrow m^2 + 4$ is prime. **[1]**

(b) By considering separately the case when n is odd and the case when n is even, prove that the following statement **is** true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4. **[4]**

6



The diagram shows triangle ABC, with $AB = x$ cm, $AC = y$ cm and angle $BAC = 60°$. It is given that the area of the triangle is $(x+y)\sqrt{3}\,\text{cm}^2$.

(a) Show that $4x + 4y = xy$. **[2]**

When the vertices of the triangle are placed on the circumference of a circle, AC is a diameter of the circle.

(b) Determine the value of x and the value of y. **[4]**

7 **(a)** Write down an expression for the gradient of the curve $y = e^{kx}$. **[1]**

(b) The line L is a tangent to the curve $y = e^{\frac{1}{2}x}$ at the point where $x = 2$.

Show that L passes through the point (0, 0). **[4]**

(c) Determine the coordinates of the point of intersection of the curves $y = 3e^{x}$ and $y = 1 - 2e^{\frac{1}{2}x}$. **[6]**

Total Marks for Question Set 3: 50